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Book Review: *Noise Induced Transitions, Theory, and Applications in Physics, Chemistry, and Biology*

Noise-Induced Transitions, Theory, and Applications in Physics, Chemistry, and Biology. By W. Horsthemke and R. Lefever. Springer, Berlin, 1984.

This is a monograph on the mathematical techniques required to understand an important new class of phenomena that can only occur in nonequilibrium, thermodynamically open, nonlinear systems interacting with a randomly fluctuating environment. The phenomena are of fundamental interest because the fluctuations, in addition to disorganizing the system as one would expect, also tend to impose a structure that is not present when the fluctuations are absent. Thus, the use of the name "Noise-Induced Transitions" for the nonequilibrium transition phenomena. In Chapter 1 the authors clearly distinguish between this type of fluctuation-induced organization and that of "dissipative structures." The latter are a consequence of the boundary conditions of a nonlinear thermodynamic system being held fixed so as to maintain a constant flux through the system to feed and sustain the dissipative structure. Therefore, the coupling between the system and the environment is deterministic in this latter case (excluding internal fluctuations which are presumed to not play a role in either organizing mechanism). The organized states considered by the authors arise out of the fluctuations in the boundary conditions and are quite different from dissipative structures. Fluctuation-induced organization has been experimentally observed in such systems as an electrical parametric oscillator, open chemical systems (Briggs–Rauscher reaction) and thermoluminescence to mention a few of those considered in the monograph.

To understand this phenomenon, one must examine the interplay between the nonlinearities in a system and the state-dependent fluctuations produced by the coupling of the system to the environment. The authors do this admirably by giving exhaustive physical interpretations of the stochastic differential equations, so that the scientists unfamiliar with these mathematical methods can follow the line of reasoning and appreciate the significance of the various terms in the mathematical models. The authors

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have minimized the mathematical complications by restricting the study to spatially homogeneous, one-variable nonlinear systems described by a rate equation driven by fluctuations that are a product of a state-dependent deterministic function and a state-independent fluctuating function. The environment is assumed to not respond to the system and all the equations are phenomenologically motivated. Even with these restrictions, the discussions still often depend on subtle mathematical points which require careful reading.

In Chapter 2 there is a physically motivated presentation of probability theory in which the concepts of stochastic processes and probability densities are introduced. These concepts are applied to the developing of models of the coupling between systems and the environment in Chapter 3. In this chapter and in Chapter 4, the physical and mathematical properties of the noise are discussed, e.g., the effect of a finite correlation time, the meaning of white-noise, the properties of a Markov process, why this noise is Gaussian, when one can use the Fokker–Planck equation, etc. In Chapter 5 the interpretation of a stochastic differential equation in terms of stochastic integrals using the Itô and Stratonovich calculi are compared and contrasted. This chapter is quite technical and may be avoided by the less mathematically interested, although the authors do attempt to make the results accessible to the physical scientist unfamiliar with stochastic differential equations. In Chapter 6, the first detailed applications of the mathematical apparatus are made by studying the change in the most probable states of a system due to certain kinds of environmental fluctuations, e.g., the Verhulst equation driven by a fluctuating growth parameter and a model of genetic selection driven by a nonlinear multiplicative fluctuation, are analyzed. This is followed up in Chapter 7 with further examples from physics, chemistry, and biology and comparisons between theoretical predictions and experimental observations are made where the data is available. It is in this chapter, where the data show the existence of these fluctuation-induced states and the theory does a credible job of explaining the data, that one is rewarded for learning the mathematical techniques of the earlier chapters.

In Chapter 8, the effect of a finite correlation time in the fluctuations (colored noise), i.e., a non-Markov process, on the system evolution is considered. The solution technique used by the authors is to expand the phase space and treat the colored noise as an Ornstein–Uhlenbeck (OU) process, the joint process (system + OU noise) is then Markovian. A systematic perturbation expansion of the joint probability density in powers of the correlation time is developed to solve the Fokker–Planck equation for this model system. The authors then go back and reexamine some of the earlier theoretical results to find the effect of the finite correlation time on the type and location of the fluctuation induced states. There is also some application made to systems with nonlinear external noise, for example, liquid crystals. An exactly solvable model of colored noise using a dichtomous Markov process is presented in Chapter 9.

I would strongly recommend this monograph to scientists interested in learning about the phenomenon of fluctuation-induced phase transitions. In addition, I would recommend it as a text from which the statistical style of thought in the physical sciences can be taught, at least from the point of view of stochastic differential equations.

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